Multigroup Models

Overview

1. Introduction to multigroup

2. *lavaan* Example

3. *piecewiseSEM* Example

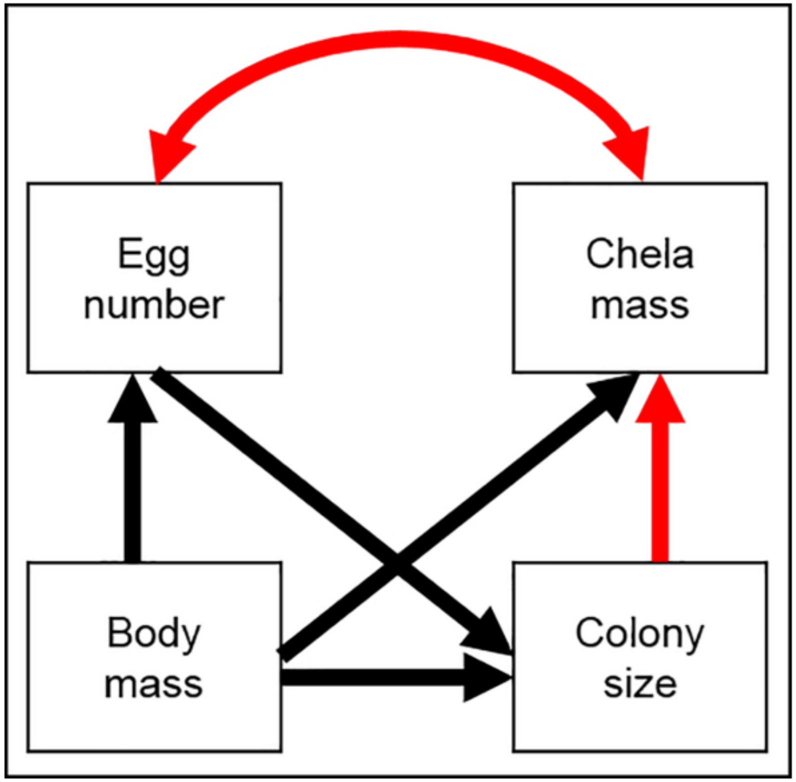
1.1 Introduction

1.1 Multigroup Models.

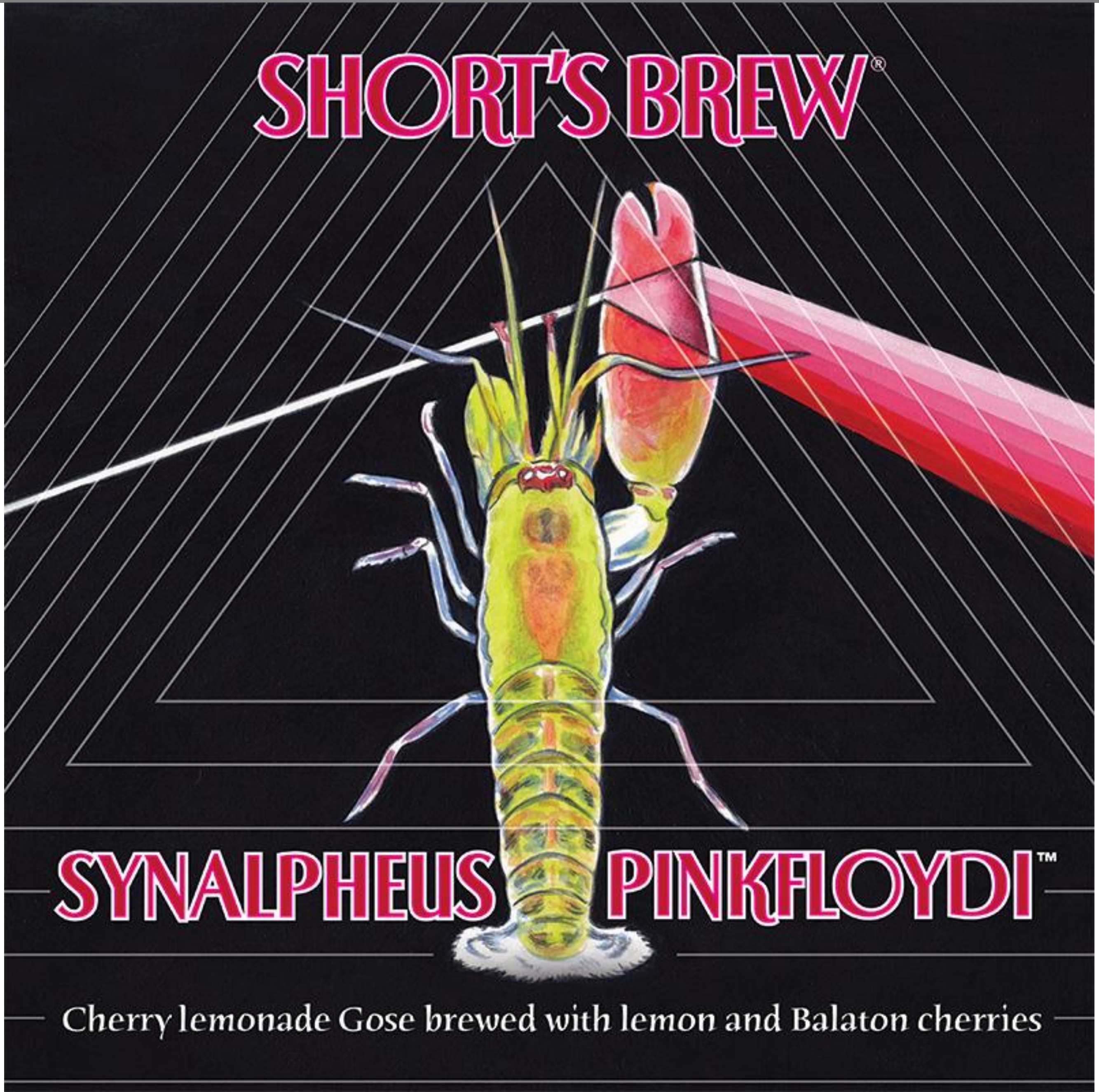
- Ask whether the effects (path coefficients) vary or are the same based on some grouping factor

- If not, figure out why:
  - Sequential addition of constraints until you break the model
  - Sequential removal of constraints until fit is achieved

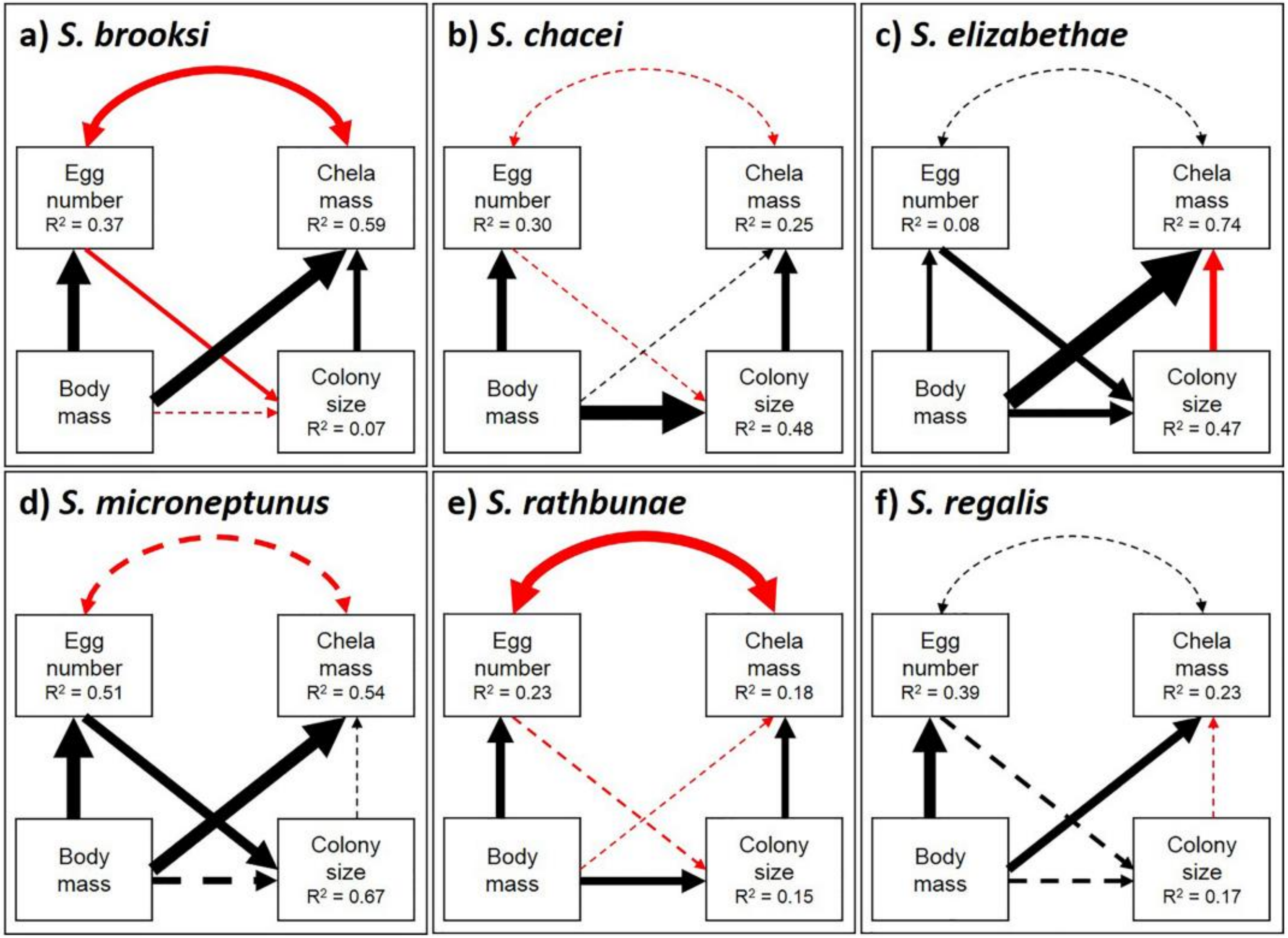
1.1 Multigroup Models. *Synalpheus* eusociality



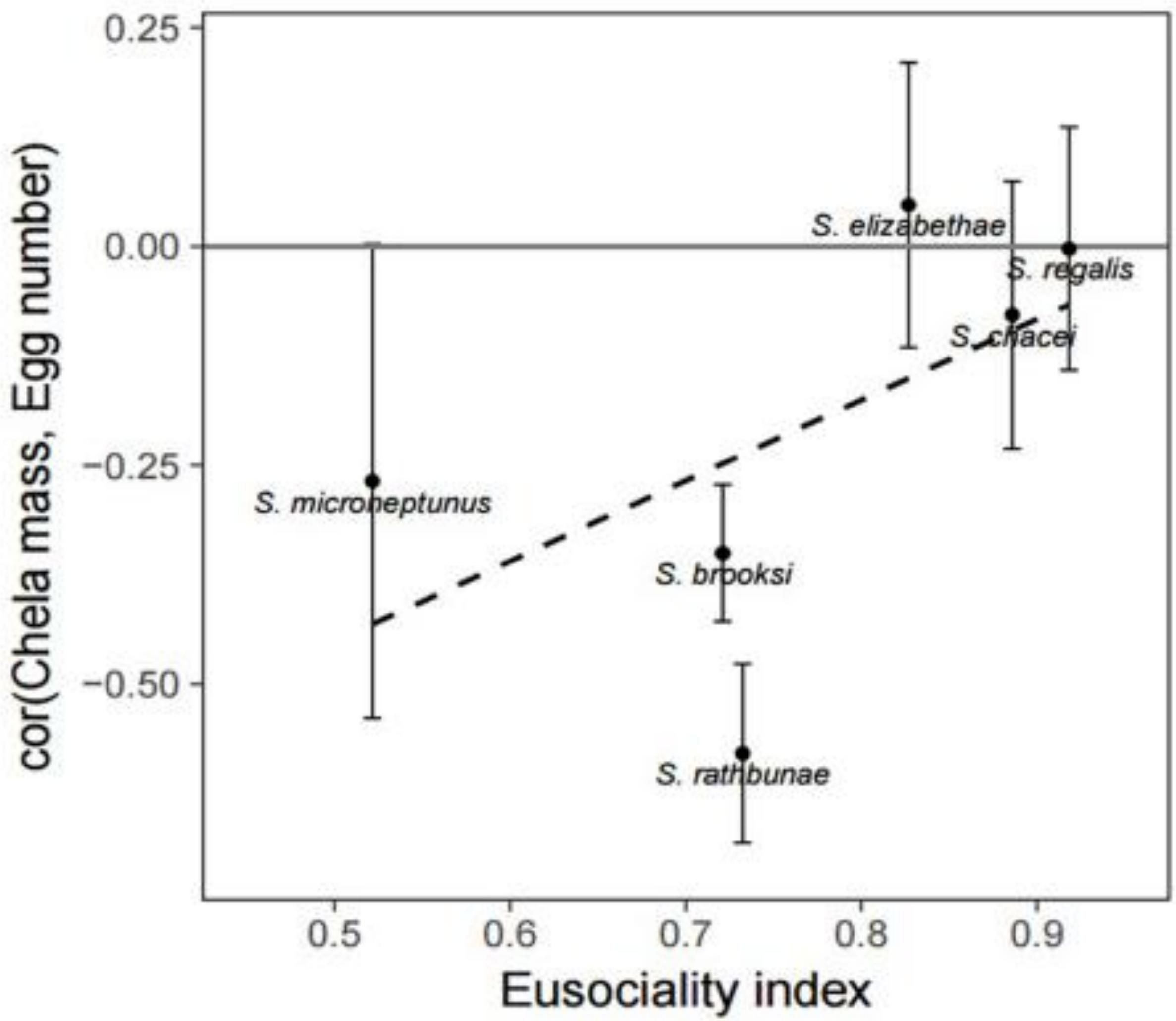
1.1 Multigroup Models. *Synalpheus* eusociality



1.1 Multigroup Models. *Synalpheus* eusociality

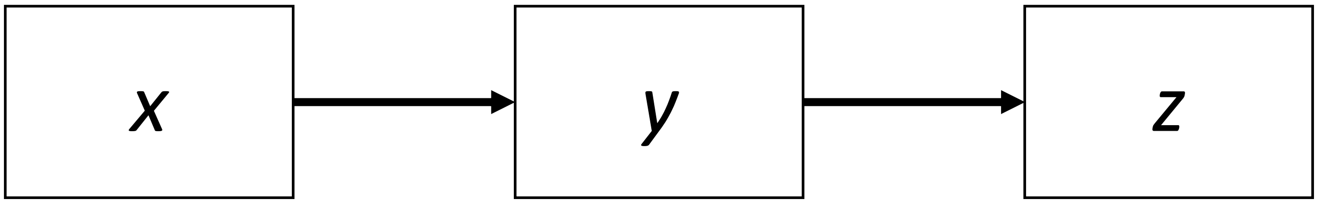


1.1 Multigroup Models. *Synalpheus* eusociality



1.1 Multigroup Models.

- Create example dataset to analyze




```
library(lavaan)
# create example dataset

set.seed(111)

dat <- data.frame(x = runif(100), group = rep(letters[1:2], each = 50))

dat$y <- dat$x + runif(100)

dat$z <- dat$y + runif(100)
```

1.1 Multigroup Models.

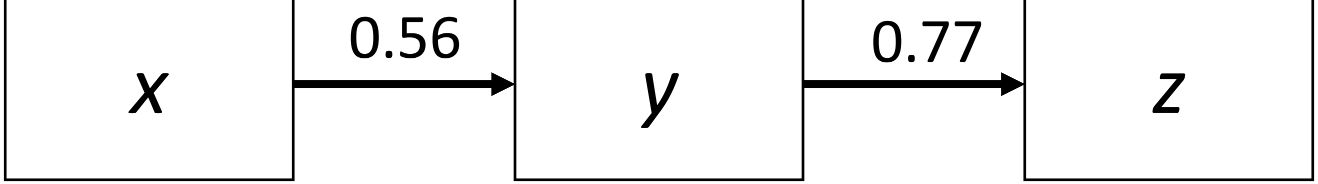
```
# create path model
multigroup.model <- '
y ~ x
z ~ y
‘

# fit path model where all coefficients vary by group
multigroup1 <- sem(multigroup.model, dat, group = "group")

summary(multigroup1, standardize = T)
```

1.1 Multigroup Models.

```
lavaan 0.6-2 ended normally after 38 iterations

  Optimization method                           NLMINB
  Number of free parameters                         12

  Number of observations per group
  a                                                 50
  b                                                 50

  Estimator                                         ML
  Model Fit Test Statistic                       0.092
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.955

Chi-square for each group:

  a                                              0.049
  b                                              0.043

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard
```

1.1 Multigroup Models.

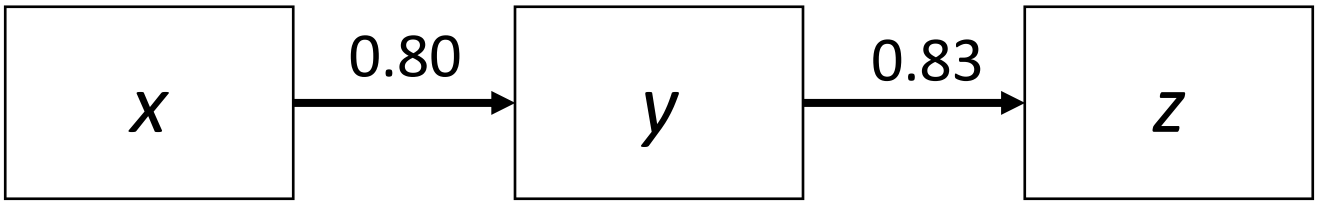
Group A




```
Group 1 [a]:

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  y ~
    x                 0.771    0.163    4.734    0.000    0.771    0.556
  z ~
    y                 1.080    0.126    8.577    0.000    1.080    0.772
```

1.1 Multigroup Models.

Group B




```
Group 2 [b]:

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  y ~
    x                 1.240    0.135    9.182    0.000    1.240    0.792
  z ~
    y                 0.897    0.086   10.465    0.000    0.897    0.829
```

1.1 Multigroup Models.

```
# now fit model with every path constrained

multigroup1.constrained <- sem(multigroup.model, dat, group = "group", group.equal
= c("intercepts", "regressions"))

# `group.equal` argument allows you to fix intercepts and coefficients to the
global value

summary(multigroup1.constrained)
```

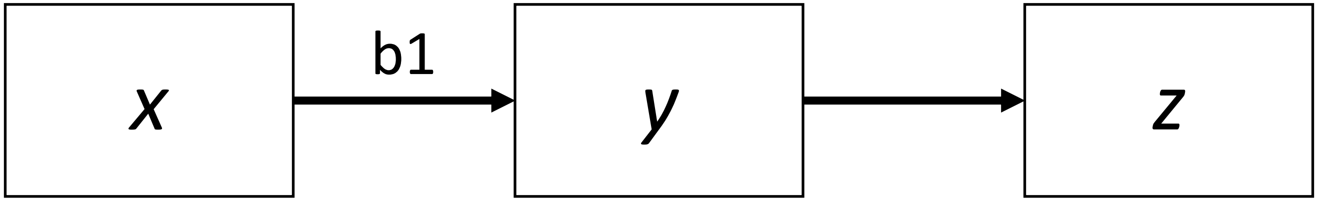
1.1 Multigroup Models.

```
Group 1 [a]:

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  y ~
    x       (.p1.)    1.046    0.108    9.678    0.000
  z ~
    y       (.p2.)    0.960    0.072   13.413    0.000



Group 2 [b]:

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  y ~
    x       (.p1.)    1.046    0.108    9.678    0.000
  z ~
    y       (.p2.)    0.960    0.072   13.413    0.000
```

1.1 Multigroup Models.

```
# compare fits
anova(multigroup1, multigroup1.constrained)

Chi Square Difference Test

                        Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
multigroup1              2 95.392 126.65 0.0921
multigroup1.constrained  6 97.251 118.09 9.9508     9.8588       4    0.04288 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

# models are significantly different = unconstrained model is better fit
# some paths differ—but which??
```

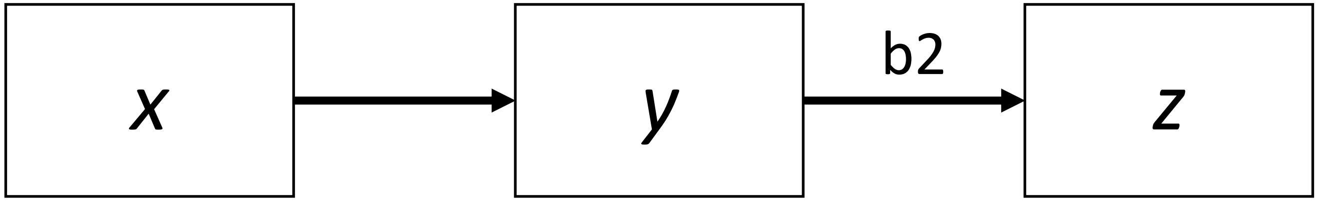
1.1 Multigroup Models.




```
# let's start by introducing a constraint
multigroup.model2 <- '
y ~ c("b1", "b1") * x
z ~ y
'

multigroup2 <- sem(multigroup.model2, dat, group = "group")

# compare the model with one constraint and free model
anova(multigroup1, multigroup2)

Chi Square Difference Test

            Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
multigroup1  2 95.392 126.65 0.0921
multigroup2  3 98.188 126.84 4.8881      4.796       1    0.02853 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Don't constrain…

1.1 Multigroup Models.




```
# repeat with the second path
multigroup.model3 <- '
y ~ x
z ~ c("b2", "b2") * y
'

multigroup3 <- sem(multigroup.model3, dat, group = "group")


Chi Square Difference Test

            Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
multigroup1  2 95.392 126.65 0.0921
multigroup3  3 94.823 123.48 1.5230     1.4309       1      0.231
```

Can constrain…

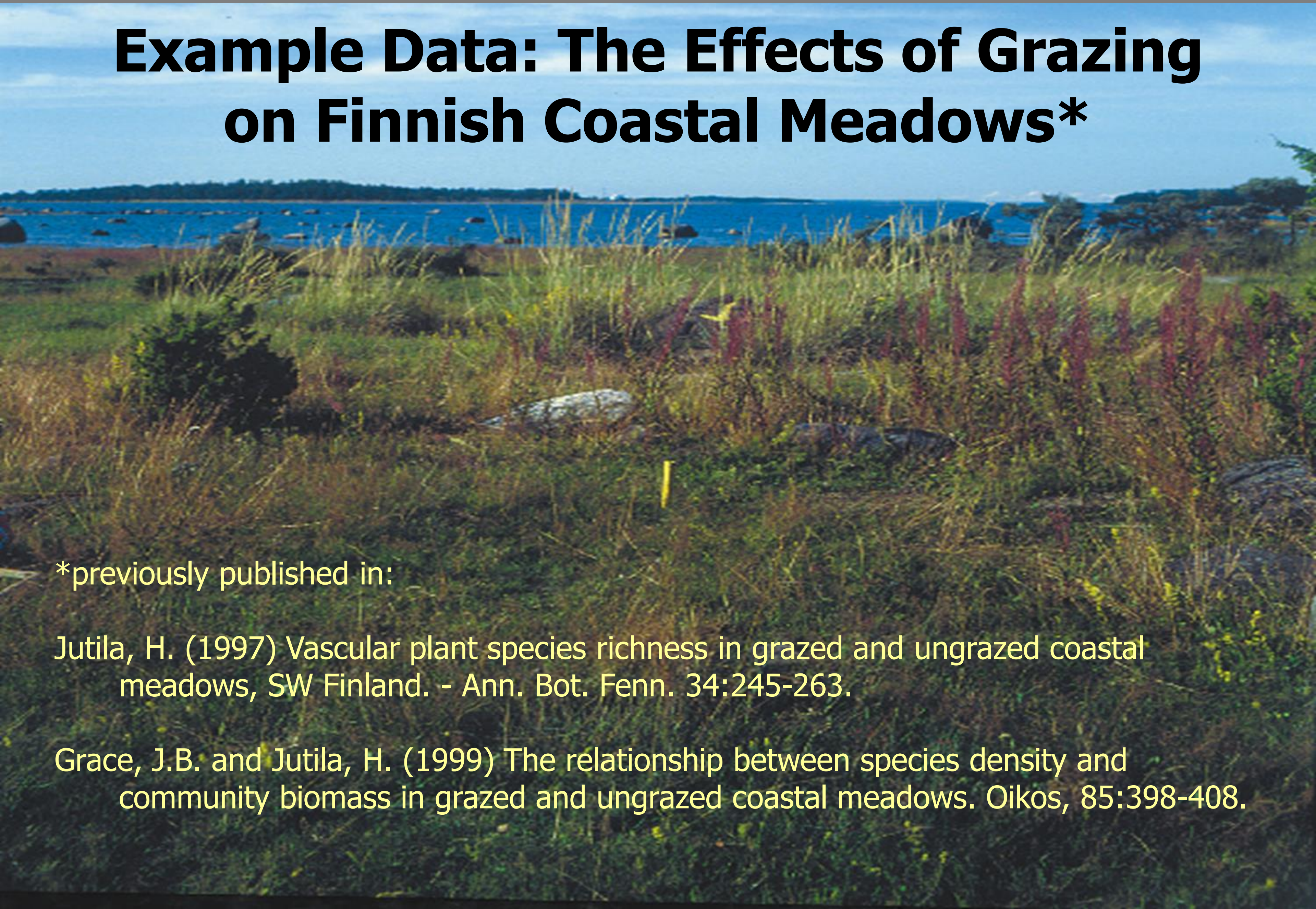
1.1 Multigroup Models.

- Best model = one path left to vary by group (x->y) while other set to global coefficient (y->z)

- Potentially very exploratory
  - Choosing which paths to constrain should be motivated by the question

- Note that standardized coefficient will still vary among groups
  - Because variances are unequal among group, coefficients must be standardized by SDs of each group regardless of whether they are constrained or not

1.2 *lavaan* Example



Example Data: The Effects of Grazing on Finnish Coastal Meadows*

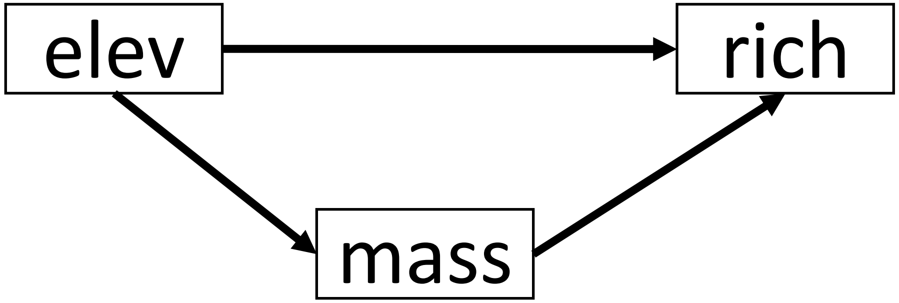
*previously published in:

Jutila, H. (1997) Vascular plant species richness in grazed and ungrazed coastal meadows, SW Finland. - Ann. Bot. Fenn. 34:245-263.

Grace, J.B. and Jutila, H. (1999) The relationship between species density and community biomass in grazed and ungrazed coastal meadows. Oikos, 85:398-408.

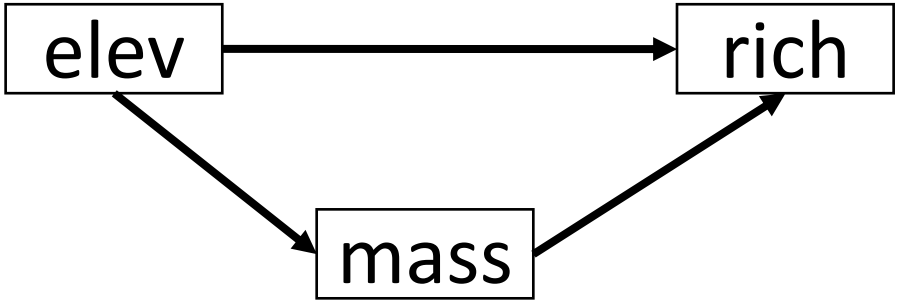
1.2 Multigroup Models. Grace & Jutila

- Interested in controls (elevation, plant biomass) on plant species' richnes in grazed and ungrazed Finnish meadows



1.2. Multigroup Models. Grace and Jutila

- Fit unconstrained and constrained model and compare the two using `anova`



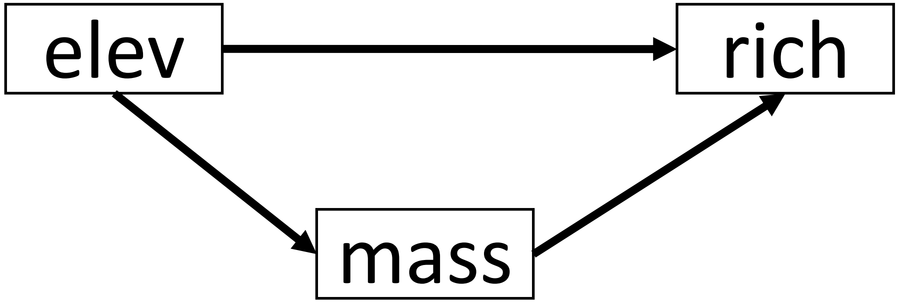
1.2. Multigroup Models. Grace and Jutila

```
Chi  Square Difference Test

                Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
jutila_lavaan    0 6666.2 6720.3  0.000
jutila_lavaan2   5 6754.4 6789.2 98.261     98.261       5  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

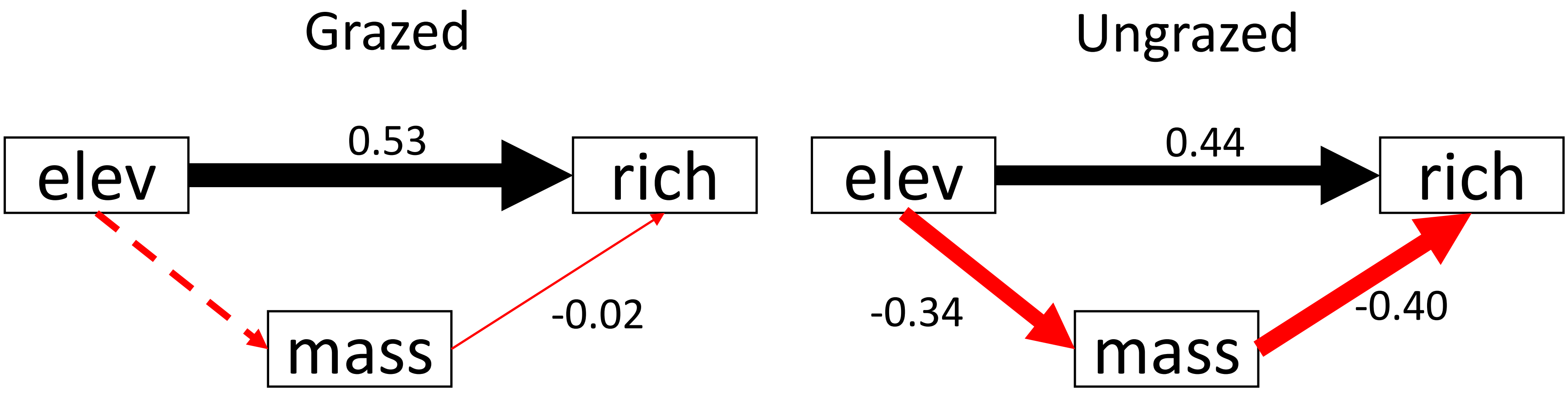
Some paths should be left to vary…

1.2. Multigroup Models. Which paths vary?

- Relax and constrain individual paths to determine which paths should vary among groups, and which should not:



1.2 Multigroup Models. Grace & Jutila



- Direct effect of elevation in grazed treatments
  - Non-significant effect of elev->mass

- Partial mediation in ungrazed treatments

- Don’t forget even though elev->rich is constrained, standardized coefficient varies based on group SD!

1.3 *piecewiseSEM* Example

1.3 Multigroup Models.

- Essentially a model-wide interaction effect

- The relationship of each variables *depends on* which group its in

- Can devise a slightly different procedure:

1) Fit interaction between every variable and group
2) For significant interactions, compute group-specific coefficient
3) For non-significant interactions, compute global coefficient
   a) If all non-significant, report pooled model
4) Return group coefficients, indicating which paths are constrained
   a) Standardized by group SD

1.3 Multigroup Models. Grace & Jutila

```
model1 <- lm(rich ~ elev * grazed + mass * grazed, meadows)

car::Anova(model1, type = "III")

Anova Table (Type III tests)

Response: rich
            Sum Sq  Df  F value    Pr(>F)
(Intercept) 3296.7   1 240.2944 < 2.2e-16 ***
elev         917.9   1  66.9057 5.401e-15 ***
grazed       220.4   1  16.0622 7.501e-05 ***
mass         429.6   1  31.3145 4.452e-08 ***
elev:grazed   1.7   1   0.9290  0.335790
grazed:mass  126.3   1   9.2052  0.002595 **
Residuals   4774.3 348
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

# constrain elev->rich path to global coefficient
# allow grazed -> rich path to vary by group
```

1.3 Multigroup Models. Grace & Jutila

```
model2 <- lm(mass ~ elev * grazed, meadows)

anova(model2)

Analysis of Variance Table

Response: mass
             Df   Sum Sq Mean Sq F value    Pr(>F)    
elev          1  2138346 2138346 58.1802 2.282e-13 ***
grazed        1   854310  854310 23.2441 2.131e-06 ***
elev:grazed   1   287419  287419  7.8201  0.005452 ** 
Residuals   350 12863853   36754                      
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

# allow elev -> mass relationship to vary by group
```

1.3 Multigroup Models.

- `multigroup` function in *piecewiseSEM* performs interaction tests and automatically returns constrained/unconstrainted coefficients based on ANOVA

```
# create piecewise version of model
jutila <- psem(
    lm(rich ~ elev + mass, data = meadows),
    lm(mass ~ elev, data = meadows)
)

# supply to multigroup
jutila.multigroup <- multigroup(jutila, group = "grazed")

# recover summary
jutila.multigroup
```

1.3 Multigroup Models. Grace & Jutila

```
Structural Equation Model of jutila.psem

Groups = grazed [ 1, 0 ]

---

Global goodness-of-fit:

  Fisher's C = 0 with P-value = 1 and on 0 degrees of freedom
```

1.3 Multigroup Models. Grace & Jutila

```
Model-wide Interactions:

  Response   Predictor Test.Stat DF P.Value
      rich elev:grazed      1.7  1  0.3358
      rich mass:grazed    126.3  1  0.0026 **
      mass elev:grazed 287418.5  1  0.0055 **

 elev -> rich constrained to the global model
```

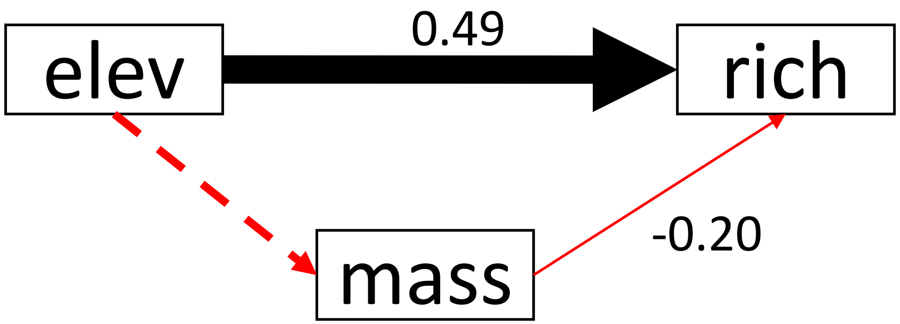
1.3 Multigroup Models. Grace & Jutila

```
Group [1] coefficients:

  Response Predictor Estimate Std.Error  DF Crit.Value P.Value Std.Estimate
      rich      elev   0.0731    0.0081 351     8.9882  0.0000       0.4967 c ***
      rich      mass  -0.0007    0.0017 162    -0.4198  0.6752      -0.0291
      mass      elev  -1.2028    0.4728 163    -2.5438  0.0119      -0.1954      *

Group [0] coefficients:

  Response Predictor Estimate Std.Error  DF Crit.Value P.Value Std.Estimate
      rich      elev   0.0731    0.0081 351     8.9882       0       0.3933 c ***
      rich      mass  -0.0072    0.0013 186    -5.4216       0      -0.3222   ***
      mass      elev  -3.2735    0.5571 187    -5.8764       0      -0.3948   ***

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05    c = constrained
```

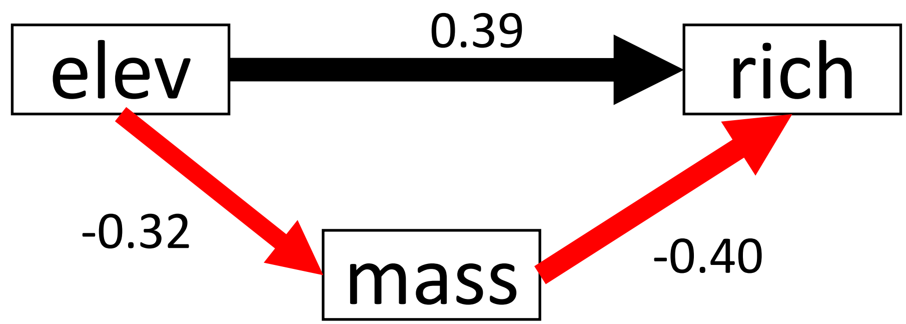
1.3 Multigroup Models.

piecewiseSEM

Grazed

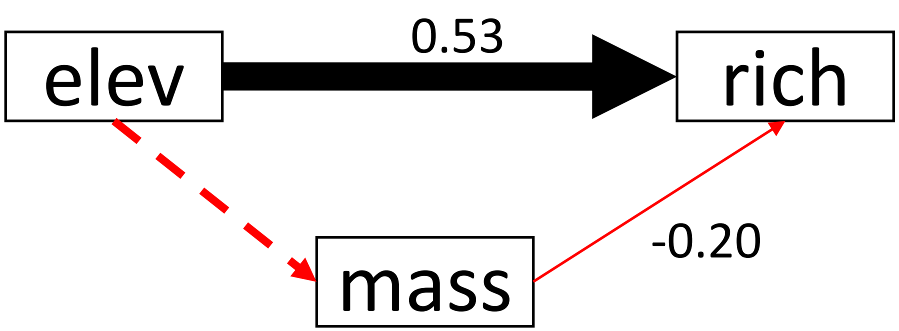


Ungrazed

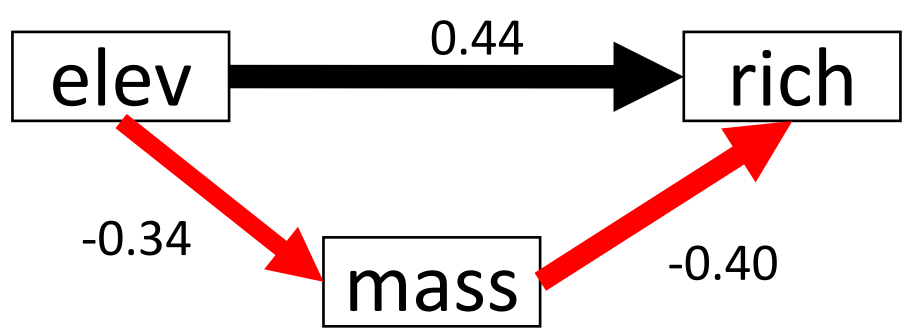


lavaan

Grazed



Ungrazed



1.3 Multigroup Models.

- *piecewiseSEM* approach reduces user control over constraints
  - Good or bad depending on mode of SEM & *a priori* knowledge
  - No way to implement coefficient fixing in *piecewiseSEM* (Shipley & Douma, in press)

- Need to have sufficient sample size to fit model to each group (or to estimate interaction terms)

- Disparate results can be produced by different groups encompassing different ranges of variability.